

Entering AP Calculus BC Summer Assignment

Welcome to AP Calculus BC!!! I am so excited to spend our year together!!! .

Directions.

- ♥ This assignment will be collected on the 1st day of classes. Exact Date TBD.
- ♥ Your summer work this year is a series of AB topics which you will review through Circuits. We have done a lot of these.....sort of a seek and solve on paper. These are great because they are self - checking!!!
- ♥ Work must still be organized and labeled. You may work on this document or you may use your own paper. Organized work is very important on the AP Exam given each May. Please take extra care to make sure that your work is easy to follow and your answers are easy to read. I will be grading the work also not just the answers.
- ♥ This assignment will count as a Quiz Grade for Quarter 1.
- ♥ I do respond to email over the summer!!! If you every have a question or need a hint or two...please reach out.....I will respond in a timely manner.

I am really looking forward to working together next school year.....have a great summer.

Be safe and stay well.



Mrs. Weber

AP Calculus -- Derivatives Circuit Training

Name _____

Beginning in the first cell, find $\frac{dy}{dx}$. Hunt for your answer, mark that cell #2 and find the next derivative. Proceed in this manner until you complete the circuit. Make sure you truly understand how each equation connects to its derivative, and that you aren't just using process of elimination. ☺

<p>Ans: 0 # <u>1</u> $y = \frac{1}{2}x^4 + 3x^2 - x + e$</p>	<p>Ans: $\frac{dy}{dx} = x(2\cos x - x\sin x)$ # _____ $y = \cos(x^2)$</p>
<p>Ans: $\frac{dy}{dx} = \sec^2 x e^{\tan x}$ # _____ $y = \ln x + e^x$</p>	<p>Ans: $\frac{dy}{dx} = \frac{3}{2} \sec^3\left(\frac{x}{2}\right) \tan\left(\frac{x}{2}\right)$ # _____ $x^3 - y^3 = 3$</p>
<p>Ans: $\frac{dy}{dx} = \frac{1}{(3-x)^2}$ # _____ $y = x^2 \cos x$</p>	<p>Ans: $\frac{dy}{dx} = \frac{3-2x-y}{x+2y}$ # _____ $x^2 y + y^2 x - 2x = 7$</p>

Ans: $\frac{dy}{dx} = \frac{1+xe^x}{x}$

_____ $y = \ln(3x^2 - x)$

Ans: $\frac{dy}{dx} = 2x^3 + 6x - 1$

_____ $y = -\frac{3}{4}x^4 - 4x^{-2} + \sqrt{x}$

Ans: $\frac{dy}{dx} = e^x - 2xe^{x^2}$

_____ $y = e^{\tan x}$

Ans: $\frac{dy}{dx} = \frac{2-y^2-2xy}{x^2+2xy}$

_____ $\sin y + \cos x = 1$

Ans: $\frac{dy}{dx} = -2x\sin(x^2)$

_____ $y = \sqrt{3x - x^2}$

Ans: $\frac{dy}{dx} = \frac{1}{x-1} - \frac{8x}{4x^2-3}$

_____ $y = \tan^{-1}x$

Ans: $\frac{dy}{dx} = -1$

_____ $y = \frac{x}{3x-x^2}$

Ans: $\frac{dy}{dx} = \cot x$

_____ $y = \ln e^x$

Ans: $\frac{dy}{dx} = \frac{x^2}{y^2}$

_____ $x^2 + xy + y^2 = 3x$

Ans: $\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}$

_____ $y = (1 + \sin^2 x)^2$

Ans: $\frac{dy}{dx} = \frac{1}{1+x^2}$

_____ $y = 4\sin^2 x + 4\cos^2 x$

Ans: $\frac{dy}{dx} = \frac{6x-1}{3x^2-x}$

_____ $y = \ln(\sin x)$

Ans: $\frac{dy}{dx} = -4\cos^3 x \sin x$

_____ $y = \sec^3\left(\frac{x}{2}\right)$

Ans: $\frac{dy}{dx} = \frac{3-2x}{2\sqrt{3x-x^2}}$

_____ $y = \frac{x^2+1}{x^2-1}$

Ans: $\frac{dy}{dx} = \frac{\sin x}{\cos y}$

_____ $y = e^x + e - e^{x^2}$

Ans: $\frac{dy}{dx} = -3x^3 + \frac{8}{x^3} + \frac{1}{2\sqrt{x}}$

_____ $y = \frac{3x-x^2}{x}$

Ans: $\frac{dy}{dx} = 4\sin x \cos x + 4\sin^3 x \cos x$

_____ $y = (1 - \sin^2 x)^2$

Ans: $\frac{dy}{dx} = 1$

_____ $y = \ln\left(\frac{x-1}{4x^2-3}\right)$

Circuit Training – Implicit Differentiation

Name _____

Directions: Beginning in cell number 1, calculate either the first or second derivative as indicated. To advance in the circuit, locate your answer and call that cell number 2. Continue in this manner until you complete the circuit.

NOTE: Attach additional pages as necessary to clearly communicate the calculus. I expect to see a lot of work!

<p>Answer: undefined # <u>1</u> Find $\frac{dy}{dx}$ for the circle $x^2 + y^2 = 25$.</p> <p>To advance in the circuit, evaluate $\frac{dy}{dx}$ for the point (3, -4).</p>	<p>Answer: $-\frac{2}{\sqrt{3}}$ # _____ Find $\frac{d^2y}{dx^2}$ for the circle $x^2 + y^2 = 25$.</p> <p>To advance in the circuit, evaluate $\frac{d^2y}{dx^2}$ for the point (3, -4).</p>
<p>Answer: $-\frac{25}{4}$ # _____ Find the slope of the tangent line to $\cos(\pi x) = x^7 y^2$ at the point (-1, 1).</p>	<p>Answer: 1 # _____ For the circle centered at the origin with radius 4 and equation $x^2 + y^2 = 16$, find $\frac{dx}{dt}$ at the first quadrant point where $x = 2$ if $\frac{dy}{dt} = -3$ at that instant.</p>
<p>Answer: $\frac{16}{3}$ # _____ If $\sin y + x = \frac{7}{2}$, find the rate of change at the point $(3, \frac{\pi}{6})$.</p>	<p>Answer: $\frac{1}{8}$ # _____ The relation $y^2(4 - x) = x^2$ has a slope of _____ when $x = 3$ and $y = -3$.</p>
<p>Answer: $\frac{1-3y}{3x+2y}$ # _____ Calculate the slope of the tangent line to $x^2 - xy + y^2 = 19$ at the point (2, 5).</p>	<p>Answer: $3\sqrt{3}$ # _____ Find $\frac{dy}{dx}$ for $3\sqrt[3]{x} - 12\sqrt[3]{y^4} = 9$</p>

<p>Answer: $\frac{1}{16\sqrt[3]{x^2y}}$</p> <p># _____ Find $\frac{dy}{dx}$ for $\tan(xy) = x + y$.</p>	<p>Answer: $\frac{3}{4}$</p> <p># _____ Calculate $\frac{dy}{dx}$ for the relation $3x + xy = y$.</p> <p>To advance in the circuit, find the instantaneous rate of change at the point $(\frac{1}{4}, 1)$.</p>
<p>Answer: $\frac{25}{64}$</p> <p># _____ Find $\frac{dy}{dx}$ for the hyperbola $x^2 - y^2 = 16$.</p>	<p>Answer: $\frac{7}{2}$</p> <p># _____ Find $\frac{dy}{dx}$ for $y^2 = \frac{1}{2x+5}$.</p>
<p>Answer: $-\frac{5}{2}$</p> <p># _____ For the relation $\sqrt{x+y} = 3x$, find the value of x for which $\frac{dy}{dx} = 17$ when $y = 8$.</p>	<p>Answer: $\frac{-1}{y(2x+5)^2}$</p> <p># _____ Find the slope of the tangent line to the ellipse $x^2 + 4y^2 = 16$ at the point $(4, 0)$.</p>
<p>Answer: $\frac{1-y\sec^2(xy)}{x\sec^2(xy)-1}$</p> <p># _____ Write the equation of the line tangent to $x^2 + y^2 = 25$ at the third quadrant point where $x=-3$.</p> <p>To advance in the circuit, find the y-intercept of the tangent line.</p>	<p>Answer: $\frac{x}{y}$</p> <p># _____ Given the relation $x + 3xy + y^2 = 2x$, find $\frac{dy}{dx}$.</p>

Circuit Training – What f' and f'' tell you about f .

Name _____

Directions: Beginning in cell #1, answer the question based on your analysis of f' and/or f'' . Understanding general graph behavior will be useful too. NO TECHNOLOGY is needed for any of these questions. Hunt for your answer; that becomes problem #2. Continue in this manner until you finish.

Answer: $(-\infty, -2) \cup (2, 3)$ # 1 Given the function $f(x) = -x^2 + 7x + 6$. Find the interval(s) in which $f(x)$ is decreasing.Answer: $(-2, 0)$ # _____ Find the interval(s) on which $h(x)$ is concave down if $h'(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 9x + 1$.Answer: $(\frac{7}{2}, \infty)$ # _____ The maximum value for the function $f(x) = -2x^2 - 12x - 11$ is _____.Answer: $(0, 2)$ # _____ Given the rational function $h(x) = \frac{1}{x^2-9}$; find the intervals on which $h(x)$ is increasing.

Answer: $(\frac{1}{2}, \infty)$

_____ If $f'(x) = 4x^3 - 2x^2$, then $f(x)$ is concave down on what interval(s)?

Answer: $(-\infty, -1)$

_____ Find the interval on which $f(x) = x\sqrt{x+3}$ is increasing and concave up.

Answer: $(-\infty, -3) \cup (-3, 0)$

_____ Find the y-value of the local maximum for $f(x) = x^3 + 4x^2 - 3x - 5$.

Answer: 7

_____ If $f'(x) = 4x^3 - 2x^2$, then $f(x)$ is increasing on what interval(s)?

Answer: 1

_____ The graph of $g(x) = |x^2 - 4|$ is concave down and decreasing on the interval _____.

Answer: $(0, \frac{1}{3})$

_____ Find the x-coordinate of the inflection point (if any) of $f(x) = x^3 - \frac{1}{2}x^2 + 4x + 1$.

Answer: $(-2, \infty)$

_____ Find the interval(s) on which $g(x) = x^2e^x$ is decreasing.

Answer: $(-\frac{3}{2}, 3)$

_____ Given $g(x) = x + \sin x$. Find the interval(s) on which $g(x)$ is decreasing.

Answer: $(-3, -2)$

_____ Find the interval on which $f(x) = \ln(x^2 + 1)$ is decreasing and concave down.

Answer: 13

_____ Where is $f(x)$ concave up given that $f''(x) = -x^3 + 3x^2 + 4x - 12$?

Answer: none

_____ The graph of $y = |\ln x|$ has an absolute minimum value where $x =$ _____.

Answer: $\frac{1}{6}$

_____ Find the interval(s) on which $f(x) = x\sqrt{x+3}$ is decreasing.

Circuit Training – U-Substitution (Indefinite Integrals)

Name _____

Directions: Beginning in cell #1, evaluate the indefinite integral. Search for your answer. When you find it, that cell becomes #2. Work that problem and then hunt for the answer. Continue in this manner until you complete the circuit. You should not need technology to evaluate these integrals! Do not guess at the end! Really work them out and then check for your answer!

Answer: $-\frac{1}{4}(4-x)^4 + C$

1 $\int 2x(x^2 + 3)^3 dx$

Answer: $-\frac{1}{4}\cot^4 x + C$

_____ $\int \frac{1}{\sqrt{1-4x^2}} dx$

Answer: $\ln(x^2 + 3) + C$

_____ $\int \cos\left(\frac{x}{2}\right) dx$

Answer: $\frac{1}{4}(x^2 + 3)^4 + C$

_____ $\int (4x + 3)\sqrt{2x^2 + 3x} dx$

Answer: $\frac{1}{2}\sin^2(2x) + C$

_____ $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

Answer: $\frac{2}{3}(2x^2 + 3x)^{\frac{3}{2}} + C$

_____ $\int \frac{2x}{x^2+3} dx$

Answer: $-e^{\cos x} + C$

_____ $\int \frac{\ln(x+4)}{x+4} dx$

Answer: $\frac{1}{2} \sin^{-1}(2x) + C$

_____ $\int \frac{1}{4+x^2} dx$

Answer: $-\ln|\cos x| + C$

_____ $\int x \tan(x^2) dx$

Answer: $2\sqrt{2 + e^x} + C$

_____ $\int \sec^2\left(\frac{x}{4}\right) dx$

Answer: $-2 \cos \sqrt{x} + C$

_____ $\int \frac{\tan^{-1}x}{1+x^2} dx$

Answer: $\frac{1}{2} \sec(2x) + C$

_____ $\int x e^{x^2+1} dx$

Answer: $\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$

_____ $\int \frac{e^x}{\sqrt{2+e^x}} dx$

Answer: $\frac{1}{2} (\ln(x+4))^2 + C$

_____ $\int \frac{6x^2+4x+2}{x^3+x^2+x} dx$

Answer: $2 \sin\left(\frac{x}{2}\right) + C$

_____ $\int \sec(2x) \tan(2x) dx$

Answer: $2 \ln|x^3 + x^2 + x| + C$

_____ $\int (4 - x)^3 dx$

Answer: $\frac{1}{2}(\tan^{-1}x)^2 + C$

_____ $\int \cot^3 x \csc^2 x dx$

Answer: $-\frac{1}{2} \ln|\cos(x^2)| + C$

_____ $\int 2 \sin(2x) \cos(2x) dx$

Answer: $4 \tan\left(\frac{x}{4}\right) + C$

_____ $\int e^{\cos x} \sin x dx$

Answer: $\frac{1}{2}e^{x^2+1} + C$

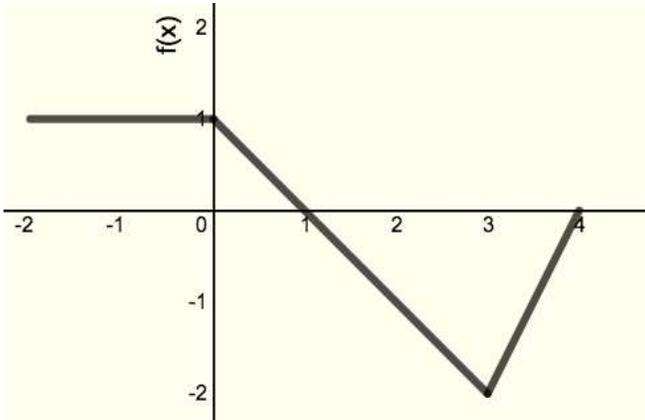
_____ $\int \frac{\sin x}{\cos x} dx$

Circuit Training – Fundamental Theorem of Calculus, Part I Name _____

Directions: Beginning in cell #1, use the Fundamental Theorem of Calculus Part I (and occasionally Part II) to answer the question. Search for your answer and that problem becomes #2. Continue in this manner until you complete the circuit.

NOTE: Any questions about the function $H(t)$ pertain to the following given information...

Let $H(t) = \int_1^t f(x)dx$ where $f(x)$ is the continuous function composed of three line segments with domain $[-2, 4]$ as graphed below:



Answer: $-\frac{5}{2}$

1 Let $F(x) = \int_0^x 5dt$. Find $F(3)$.

Answer: $\frac{2}{3}$

_____ Find $\frac{d}{dt} \int_{-1}^{\tan t} \frac{2}{1+x^2} dx$ and evaluate it for $t = -\frac{\pi}{4}$.

Answer: 1

Find $F'(\frac{3\pi}{2})$ given $F(t) = \int_5^t \frac{2x}{\pi} e^{\cos x} dx$.

Answer: -2

_____ $H''(1) = ?$

<p>Answer: -1</p> <p># _____ $H(-2) = ?$</p>	<p>Answer: 4</p> <p># _____</p> <p>$G(x) = \int_{-2}^x \cos\left(\theta + \frac{\pi}{2}\right) d\theta.$ $G'\left(-\frac{\pi}{2}\right) =$</p>
<p>Answer: 15</p> <p># _____</p> <p>Let $G(x) = \int_x^2 t dt.$ Find $G(4).$</p>	<p>Answer: -3</p> <p># _____</p> <p>Given $W(t) = \int_2^t \ln(x - 1) dx.$ Find $W''\left(\frac{5}{2}\right).$</p>
<p>Answer: 3</p> <p># _____</p> <p>The position function, $s(t),$ is defined as $s(t) = s(0) + \int_0^t \left(\frac{8}{\pi} + \sec^2 \beta\right) d\beta$ where $s(0) = -6.$ Find $s\left(\frac{\pi}{4}\right).$</p>	<p>Answer: 0</p> <p># _____ Now evaluate $H'(3).$</p>
<p>Answer: 2</p> <p># _____ The next questions are about $H(x).$ Evaluate $H(1).$</p>	<p>Answer: -6</p> <p># _____</p> <p>Let $F(x) = \int_3^x \sqrt{1 + t} dt.$ Find $F'(15).$</p>